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Mechanics of the sandglass*

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'To see a World in a Grain of Sand . . .'

William Blake, Auguries of Innocence

Abstract. The factors that control the period delineated by a sandglass have been systematically investigated. This form of interval timer (known since medieval times) depends on particle flow, which exhibits characteristics quite different from those of liquid flow. Thus, for example, the rate of flow is independent of the head of material in the reservoir, except for the last few centimetres. The particulate material need not be silica sand, but should be smooth and regular with grains of similar size: the vitreous spherical filler known as ballotini gave the most reproducible results. For a given volume of ballotini, the period is controlled by their size, the size of the orifice, and the shape of the reservoir. Provided the aperture is at least 5 $\times$ the particle diameter, the period P is given by the expression $P = KV(D - d)^{-2.5}$ where P is measured in seconds, V denotes the bulk volume of ballotini in ml, d the maximum bead diameter in mm as measured by sieve size, and D the diameter of a circular orifice in mm. The constant of proportionality K depends on the shape of the reservoir: the values for hourglass-, cone- and silo-shaped vessels were found to be 7–10, 8 and 19 respectively. The presence of a horizontal annulus around the aperture considerably extends the period by reducing the rate of flow: K is of the order of 21 for such a construction. The internal flow regimes giving rise to this behaviour were pictured with the aid of 2-D perspex models incorporating layers of coloured ballotini. The best 19 m 45 s glass exhibited a standard deviation of ± 5 s(± 0.4%), but for sandglasses in general the variation could be up to ± 1.5%. Any disturbance lengthened the period, but changes in temperature gave no observable effect within this margin of error.

Zusammenfassung. Die Faktoren, die den Zeitraum bestimmen, der von einer Sanduhr angezeigt wird, sind systematisch erforscht worden. Diese Art des Zeitnehmers (seit dem Mittelalter bekannt) basiert auf einem Teilchenfluß, der ganz andere Charakteristika aufweist als das Strömen von Flüssigkeiten. So ist zum Beispiel die Fließrate unabhängig von der Höhe des Materials im Reservoir bis auf die letzten wenigen cm. Bei dem zerkleinerten Material muß es nicht um Quarzsand handeln, aber es sollte glatt und regelmäßig mit ähnlich großen Körnern sein: das gläserne kugelförmige Füllmaterial—bekannt als Ballotini—brachte die am besten reproduzierbaren Ergebnisse: Bei einem gegebenen Volumen von Ballotini wird der Zeitraum kontrolliert von deren Größe, der Größe des Durchlasses und der Gestalt des Behälters. Angenommen die Öffnung ist wenigstens 5 mal so groß wie der Teilchendurchmesser, dann ist die Dauer P gegeben durch den Ausdruck $P = KV(D - d)^{-2.5}$ wobei P in Sekunden gemessen wird, V bezeichnet das Gesamtvolder Ballotini in ml, d den maximalen Teilchendurchmesser in mm gemessen durch die Siebgröße, und D den Durchmesser der kreisrunden Öffnungen in mm. Die Proportionalitätskonstante K hängt von der Gestalt des Behälters ab: für stundenglasähnliche, trichter- und siloartige Gefäße wurden Werte von 7–10, 8 bzw 19 gemessen. Die Anwesenheit eines horizontalen Rings rund um die Öffnung erweitert den Zeitraum beträctlich, indem er die Fließgeschwindigkeit erniedrigt: K hat ungefähr einen Wert von 21 für eine solche Konstruktion. Die internen Fließmuster, die für diese Verhalten vertantwortlich sind, wurden sichtbar gemacht mithilfe von 2-D-Plexiglas-Modellen, die zusätzlich Schichten mit farbigen Ballotini enthielten. Die beste 19m 45s Uhr wies eine Standardabweichung von $\pm$5s($\pm$0.4%) auf, aber für Sanduhren allgemein könnte die Schwankungsbriete bis zu $\pm$1.5% betragen. Jede Störung verlängert den Zeitraum, Temperaturveränderungen hatten jedoch keinen beobachtbaren Effekt innerhalb dieser Variationsbreite.

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1. Introduction

The sandglass (figure 1) is a preset interval timer dependent on particle flow. As such it is quite distinct from the clepsydra, which utilizes the controlled flow of a true liquid, generally water [1]. Instead, the sandglass has affinities with the industrially important

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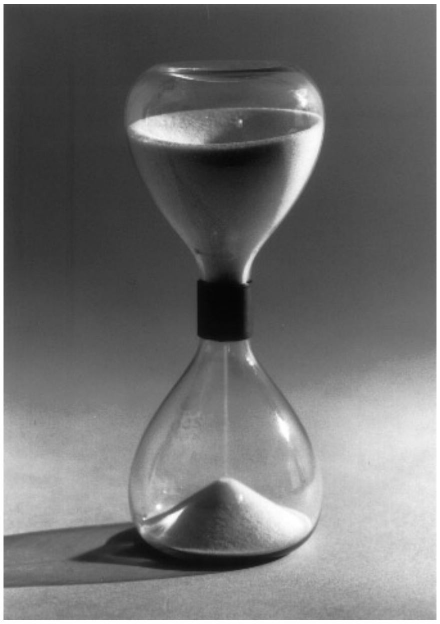

Figure 1. Working facsimile of the traditional form of sandglass.

Figure 2. The earliest known representation of a sandglass. Detail from a fresco in Siena by Ambrogio Lorenzetti, painted around 1338.

Figure 3. The oldest surviving sandglass. Probably German. (Horological Collection of the British Museum, cat. no. S 877. By courtesy of the Trustees.)

hoppers and silos [2], as well as with the formation and movement of natural sand dunes [3]. The sandglass has a surprisingly brief history, apparently dating from medieval Europe rather than being known along with the clepsydra in ancient Egypt, Greece or Rome. The first reliable pictorial representation dates from AD 1338 (figure 2), while the earliest written records are in lists of ships' stores from the same 14th century period [4, 5]. These $\frac{1}{2}$ hour glasses appear to have been used (with repeated turning) for navigation and to delimit the duration of each 4-hour watch; not until late in the 16th century were $\frac{1}{2}$ minute versions combined with a 'log' to give a ship's speed through the water [6]. From the 15th century the use of the sandglass to define the duration of sermons, lessons, manufacturing and culinary purposes, etc, became commonplace, but today only the egg-timer remains familiar. The oldest surviving sandglass, dating from 1520, is now in the British Museum (figures 3 and 4).

It will be noted that the sandglass measured out equal intervals of time—'equal hours'—and not the varying 'seasonal hours' characterizing all other civil timekeeping up to the spread of the mechanical clock in the 14th and 15th centuries.

The history, occurrence and external form of the sandglass have been examined by a few authors [7–10], but direct experimental data on its design, construction, performance and accuracy are scarce [11–13]. However, it is to be expected that economic factors attached to the bulk storage and movement of vast tonnages of particulate commodities such as sand, gravel and grain would generate far more research than the humble sandglass. There is therefore a considerable relevant literature on industrial and chemical engineering aspects of particle flow, with a noticeable growth following

Figure 4. One of the miniatures painted on the ends of the sandglass shown in figure 3: the date 1520 is within a panel of the table. In favour of its authenticity is the fact that the clocks do not incorporate a pendulum or two hands.

the introduction of the computer. Fortunately, a recent textbook [14] and an excellent review [15] provide guidance to this rapidly expanding interdisciplinary field. From this past work, it soon became apparent that many factors, some major and some minor, might affect rate of flow and so the emptying period of a sandglass. These were distinguished and investigated in as systematic a manner as possible.

2. Nature of the granular filling

Although always referred to as 'sand', many different particulate materials were in fact used in the traditional sandglass [16]. Besides natural silica sand, these included powdered marble, tin/lead oxides, and pulverized burnt eggshell. This last was particularly recommended. Historical recipes always stress that, whatever its nature, the filling must always be carefully washed, dried and sieved. However, exactly what is meant by the 'best' material is never stated. Presumably, reproducibility of the interval between inversion of the glass and its complete emptying is the major criterion, but it would also be important for example that the particles should not tend to settle and cake together during any period of disuse.

For preliminary tests, a 'standard funnel' was prepared by drawing-down a length of 20 mm ID (internal diameter) pyrex tubing to produce a jet of 3.5 mm ID. A mark was placed at a height above the orifice indicating a contained volume of 50 ml. For use, the funnel was supported vertically with the outlet resting on the bottom of a 250 ml beaker, filled to the mark with the particulate material under test, and then raised simultaneously with the starting of an electronic stopwatch. The latter was stopped as the last grains left the jet. The collected particles were carefully transferred back to the funnel for the next run, thus ensuring that a constant mass was used for each determination.

2.1. Spherical particles: 'ballotini'

Intuitively, one would expect smooth spherical particles of a vitreous nature to behave in the most consistent manner, and to be the most amenable to theoretical analysis of the flow regime. The product known as 'ballotini' consists of tiny beads of sub-spherical shape, and is available in a wide range of sizes [17]. It has already been recommended in the literature [18]. We decided to try soda glass ballotini passing a 710 μm sieve, but retained by a 600 μm aperture. The average diameter within this material (measured over 100 beads) was $550 \pm 60\,\mu$m, and it had a bulk density of $1.61\,\mathrm{g\,ml^{-1}}$. Although coarser than the 50–100 μm grain size characterizing the particles in historical sandglasses, it was thought that this larger diameter would enable the influence of various factors to be more easily distinguished and measured.

The standard funnel was filled to the mark by pouring in the ballotini in a thin continuous stream. When tapped, the column dropped about 1 mm. The mean emptying period was 18.8 ± 0.1 s.

2.2. Irregular rounded particles: sand

A 600–710 μm fraction was sieved from a coarse sand sold for the filtration of water supplies. It was of mixed mineralogical composition, but the irregular grains were generally rounded rather than sharp-edged. The sieved fraction had a bulk density of $1.48\,\mathrm{g\,ml^{-1}}$.

The standard funnel was filled to the mark in a gentle continuous stream. Upon tapping, the column fell about 5 mm. The time taken to empty under constant mass conditions showed a standard deviation of ± 1.1 s about a mean of 14.0 s. So, although emptying faster, the variation was much greater than with ballotini of the same sieve size.

2.3. Cubic particles: sugar

Granulated sugar was dried at 50 °C and then sieved to produce a 600–710 μm fraction. The crystals were of cubic habit and sharp edged: most approached cubes in shape. The bulk density of this fraction was $0.90\,\mathrm{g\,ml^{-1}}$.

The standard funnel was filled to the mark as before: upon tapping the column fell about 3 mm. The time taken to empty under constant mass conditions was 22.4 ± 0.8 s, but the flow displayed a tendency to jerk, or even cease spontaneously. Clearly the aperture was close to the minimum size for this blocky material.

It was concluded that ballotini was indeed the best material to use in a modern 'sandglass'. The chosen sieve fraction was washed by elutriation until the washings ran clear, and then a final rinse with deionized

Table 1.

Material	Bulk density ($\mathrm{g\,ml^{-1}}$)	Period (s)
Poppy seed	0.63	12.2
Ballotini	1.59	11.6
Lead shot	7.16	11.0

water was given before drying the product throughly in a warm oven and re-sieving.

2.4. Density

It is known (and will be confirmed below) that the flow of granular material through an aperture occurs via the continuous collapse of a transient 'arch' or 'dome', with the result that the rate of flow is independent of the head except at very low values of the latter. Most of the weight of the bed is transferred to, and supported by, the walls of the container. The density of particles of similar shape and size would therefore be expected to have little influence on the volumetric rate of flow, any given volume containing the same number of particles. Obviously the mass flow would be a direct function of the apparent density.

This proposition was checked by sieving 600–850 μm mesh fractions from ballotini, lead shot, and de-oiled poppyseed. All were close to spherical in shape, but differed greatly in density. Bulk (apparent) densities were determined by pouring into a tared 25 ml measuring cylinder and weighing. Each material was then poured into the standard funnel up to the 50 ml mark, and the period required for complete emptying timed repeatedly. The results are shown in table 1.

It will be seen that, for a constant volume of similarly shaped and sized particles, the discharge period is little affected by their density: this length of time was only 8% less for an 11$\times$ increase in bulk density. Indeed, it is possible that this variation is due more to differences in packing and surface texture than to changes in density.

In view of this, the historically vaunted superiority of 'Venetian sand' (a dense particulate of lead/tin oxides) for long-period sandglasses (up to 24 hours) is puzzling. Perhaps it was not its density that was important, but rather an ability to give a very fine non-caking powder that would flow steadily and reliably through a tiny aperture against the counter-current of displaced air. (It is commonly overlooked that in the traditional sealed sandglass the air displaced by the grains entering the lower reservoir has no alternative but to percolate upwards through the aperture and the particulate bed above it. A sealed water or mercury clepsydra must have a tube by-passing the orifice to carry this gas flow [19].)

3. Influence of height of a particulate column

It has already been stated that the rate of flow of particulate material is expected to be independent of its height in the reservoir, except perhaps over the last few cm. This was confirmed by catching the flow of ballotini from a 2.8 mm orifice for a 10 second period every minute, and weighing these samples. The flow was constant at $0.67\,\mathrm{g\,s^{-1}}$ throughout most of a 19 m 45 s emptying period, only dropping to $0.62\,\mathrm{g\,s^{-1}}$ during the last minute.

This represents another fundamental distinction from liquid flow.

4. Design

The classic sandglass incorporates a thin perforated metal diaphragm held between two glass ampoules of characteristic shape (figures 1, 2 and 3). Time is of course measured by the interval elapsing between inverting the glass and the last grains falling from the aperture. Given the type of granular filling to be used, factors that might be expected to control this period are the total volume of the particles, the diameter of the aperture, and the shape of the reservoir(s).

4.1. Diameter of the aperture

This parameter was the next to be investigated. A 750 ml glass mineral water bottle with a tapering neck and screw-type aluminium cap was chosen, and its bottom cut off. All but the rim of the elastomer lining the screw cap was also removed, enabling sharp-edged circular holes to be made at the centre of the thin (0.35 mm) metal. This construction imitated the style of orifice employed in the oldest group of historical sandglasses.

For tests, the modified bottle was loosely supported in an inverted vertical position with its perforated cap resting upon a plastic tray whilst the body section was loaded with the 600–710 μm ballotini. The apparatus was then raised and clamped simultaneously with the starting of the stopwatch, the stop button being pressed when the flow diminished abruptly.

4.1.1. The minimum size Because the particles have passed a 710 μm sieve it might be thought that this figure would represent the minimum size of aperture. This is not so: the flow in a sandglass or hopper is quite different from both fluid flow and agitated sieving. Since it depends on the formation of a continuously collapsing arch, an aperture several times the largest grain diameter is essential if a stable 3-D dome is never to form. It was frequently doing so with the blocky sugar crystals in the preliminary tests above, where the ratio of aperture to largest grain diameter was 4.9.

The hole in the cap restraining the flow of our standard 600–710 μm ballotini was initially made 2.0 mm in diameter. Few beads came out when the bottle was raised from the tray. The orifice was then enlarged in 0.2 mm steps: not until 2.8 mm was reached did a stable flow ensue; a ratio of aperture to mean grain diameter of 5:1, and of aperture to maximum (sieve) size of 4:1.

It was decided that, to cover most particulate materials, a minimum aperture of 5× the maximum sieve size would probably represent a safer practical limit.

4.1.2. Effect of aperture on period The cap bearing a 2.8 mm hole was left in place, and 750 g (466 ml) of standard 600–710 μm ballotini poured into the inverted bottle. The emptying period for each of 10 runs was then determined, carefully collecting, check-weighing, and re-loading the same mass each time. The mean period was 19 m 45 s, with a standard deviation of ± 5 s(± 0.4%). The orifice was then enlarged in 1 mm stages with a tapered reamer (each diameter being measured against a magnified reticle) and the timing procedure repeated until a 10 mm aperture had been reached. A plot of our results is included in figure 5. The shape of this graph suggested that the experimental results might obey a power law such that:

$$P \propto \frac{V}{D^n}$$

where P is the emptying period, V is the volume of ballotini, D is the diameter of the aperture, and n is an exponent. Rewriting, we obtain:

$$\log \frac{1}{P} = n \log D - \log V + \text{constant}.$$

That is, for a fixed value of V, a plot of $\log 1/P$ versus $\log D$ would give a straight line of slope n. When tested in this way our results gave graph I in figure 6. A very satisfactory linear plot was obtained for all but the two smallest apertures, which appeared to run a little too fast by comparison with those of greater diameter. We surmise that this may be due to less grain-on-grain jostling and mutual interference than in the thicker streams. The slope of the linear portion of the graph gave $n = 2.9$. The same exponent has been obtained by earlier researchers [20, 21] working with centimetre-size orifices and industrial rates of flow of seeds and the like, but appears rather empirical and unsatisfying. Also, our figure was found using only 600–710 μm ballotini, but quoted in isolation the formula $P \propto 1/D^{2.9}$ could give the impression that emptying period is close to being inversely proportional to the cube of the aperture diameter but is independent of particle size.

The latter is not true: 690 g of 600–710 μm ballotini fell through a 4.0 mm orifice in the bottle apparatus in 8 m 07 s; the same mass of 710–850 μm material took 58 s longer.

5. Theory

A classic paper on the flow of granular solids through orifices is that of Beverloo *et al* [21]. They were the first to apply dimensional analysis to the problem, pointing out that if it is assumed that the mass flow rate (W_t) is a function of orifice diameter (D), bulk density (ρ_B) and the acceleration due to gravity (g) only, then one may write:

$$W_t \propto D^a \rho_B^b g^c.$$

Hence, dimensionally

$$\frac{[M]}{[T]} = [L]^a \left[\frac{M}{L^3}\right]^b \left[\frac{L}{T^2}\right]^c$$

from which $b = 1$, $c = 0.5$ and $a = 2.5$, giving the idealized expression:

$$W_t \propto \rho_B \sqrt{g} D^{2.5}.$$

To bring this expression into accord with the observed exponent of D of 2.9, Beverloo *et al* proposed the introduction of an additional factor: the diameter of the particles (d) multiplied by an arbitrary dimensionless constant (k). This gave:

$$W_t \propto \rho_B \sqrt{g} (D - kd)^{2.5}. \qquad (1)$$

The term $(D - kd)$ was explained by suggesting that the fall of a particle through an aperture would be hindered if it happened to be near the edge of the hole, and might not occur at all if it were closer than its radius. The overall effect would be to reduce the effective diameter of an orifice by at least one grain diameter.

Since the period P of a sandglass is related to W/W_t, where W is the weight of particulate material sealed within it, the Beverloo formula leads to the expectation that, for a given volume of particulate filling V,

$$P \propto \frac{V}{(D - kd)^{2.5}}. \qquad (2)$$

Noting that the maximum diameter of our spherical ballotini (as measured by sieve size) was close to 0.7 mm, we plotted our values of $\log 1/P$ versus $\log(D - 0.7)$ and obtained the graph marked II in figure 6. Again the two lowest points were discrepant, but the linear portion gave a slope of 2.5. We therefore propose that, for near-spherical particles:

$$P \propto \frac{V}{(D - d)^{2.5}} \qquad (3)$$

where d is the maximum particle diameter and $D > 5d$. Alternatively, $k = 1.3$ in formula (2) if the mean diameter of the particles is employed. Beverloo *et al* proposed $k = 1.4$ to encompass the variety of shapes found in a range of vegetable seeds.

5.1. General formulae

Equation (3) may be written:

$$P \propto K \frac{V}{(D - d)^{2.5}} \qquad (4)$$

where K is the constant of proportionality applying to unit volume. Thus:

$$\log\left(\frac{1}{P}\right) = 2.5 \log(D - d) - \log K. \qquad (5)$$

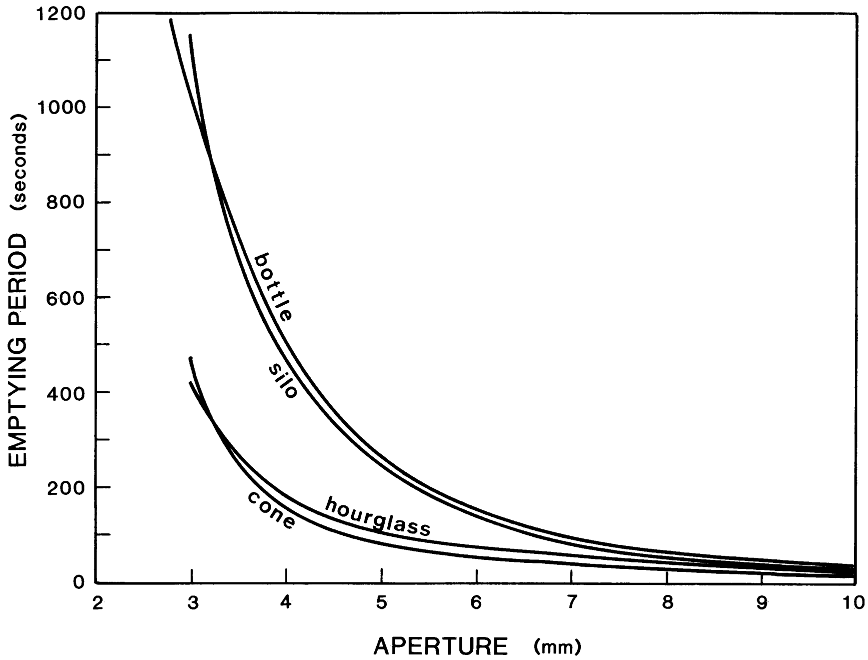

Figure 5. Arithmetic plot of emptying period versus diameter of aperture.

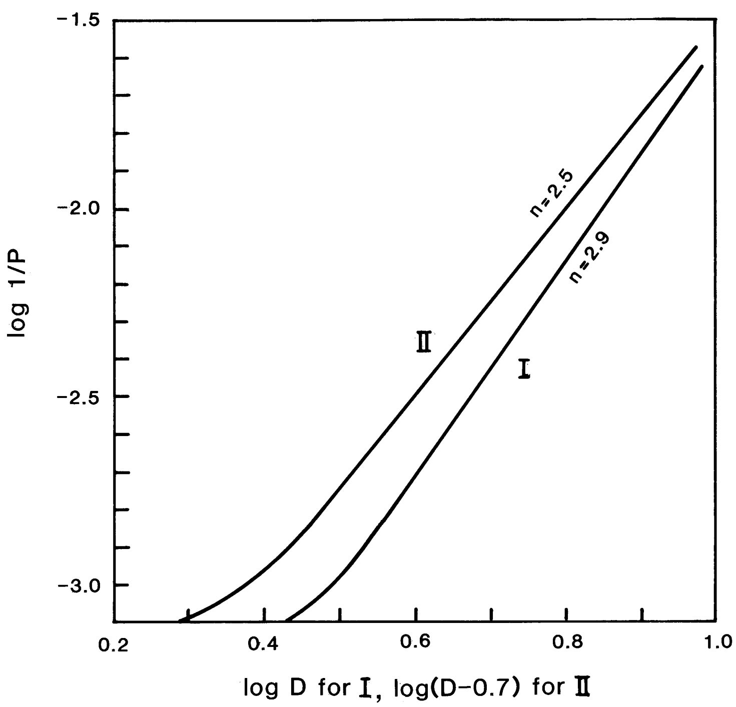

Figure 6. Logarithmic plot of the data of figure 5.

Hence a plot of $\log(1/P)$ versus $\log(D-d)$ should be a straight line of slope 2.5 and intercept equal to $-\log K$.

The periods obtained with the 'bottle' apparatus above

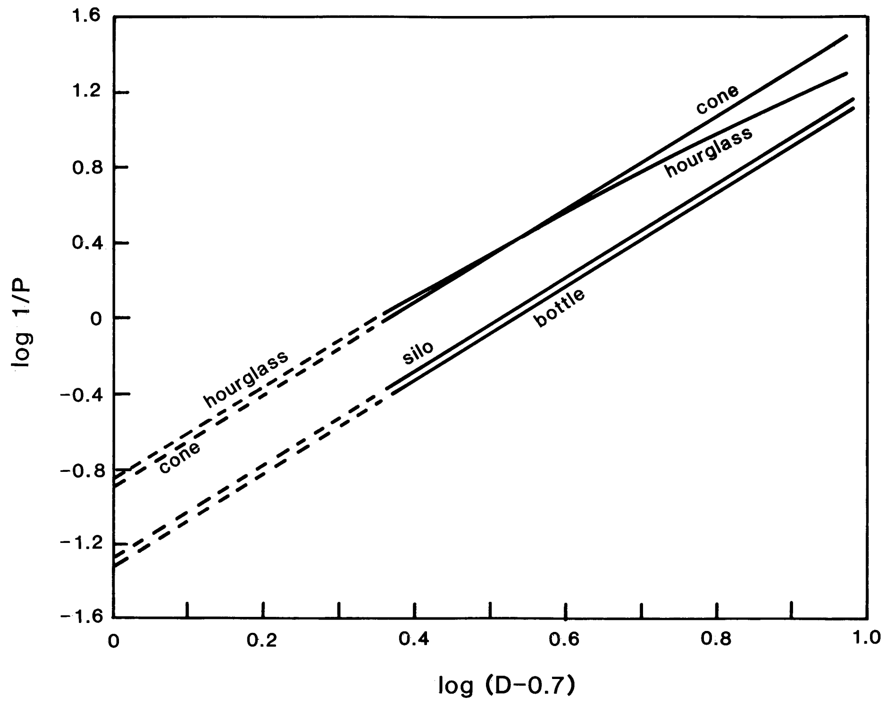

Figure 7. Plots of $\log(1/P)$ versus $\log(D-d)$. The dashed lines are extrapolations.

were reduced to those corresponding to unit volume of ballotini by dividing by 466, and plotted in the form of equation (5) (see figure 7). A straight line graph was indeed obtained, and its extrapolated intercept gave $K = 21$. So, for a bottle-shape vessel with an annular-type circular orifice, and containing spherical or near-spherical particles, the period is given by:

$$P = 21\frac{V}{(D-d)^{2.5}} \tag{6}$$

where P is measured in seconds, V is the volume of ballotini in ml, D is the diameter of the orifice in mm, and d is the maximum bead diameter as measured by sieve mesh. Alternatively, a value for d of 1.3× the mean diameter may be employed. (d equalled 0.7 mm with our standard filling material.)

6. Shape of the reservoir

The literature suggested that the shape of the reservoir could be another factor that affected the rate of flow of particles emerging from it, and so the overall emptying period. To investigate this, the 'bottle' apparatus was supplemented with specially constructed glass vessels of conical, 'silo' (a cylinder terminated by a shallow 112° cone) and traditional 'hourglass' shape. (Figures 9 and 10 incorporate these outlines.) All were of nominal 500 ml capacity, and were fibreglassed on the outside for the lowermost 10 cm to give support to the thin glass jets. By grinding back on a diamond wheel, the orifices were enlarged in 1 mm steps from 3.0 to 10.0 mm diameter without producing an annular 'shelf' surrounding the exit hole. 750 g (466 ml) of our standard ballotini was used in all the timed tests.

The results are included in figure 5. It will be seen that the shape of the reservoir is indeed an important factor, 'bottle' and 'silo' forming one similar pair, and 'cone' and 'hourglass' another pair. The latter, with smooth stepless entries into the exit apertures, ran much faster than the designs where a horizontal (or near-horizontal) annulus surrounded the hole. The periods of the corresponding sandglasses were affected in a reciprocal manner. The effect was quantified by plotting $\log(1/P)$ for unit volume against $\log(D-0.7)$, as shown in figure 7. The silo, like the bottle, is of constant diameter for much of its volume, and the cone varies regularly with height: all three gave linear plots. The hourglass, however, first expands and then contracts with distance down its axis, so it is perhaps not surprising that this shape gave a shallow curve. The relative discharge rates and emptying periods were reflected in the values found for K by extrapolating downwards and noting the intercept: see table 2.

Therefore, cylindrical silo-shaped reservoirs separated by a horizontal annular diaphragm pierced with a minimal aperture should give the longest period sandglass for a given volume of a certain particulate filling.

Table 2.

Type	Characteristics	K
‘Bottle’	Orifices surrounded by horizontal annulus	21
‘Silo’	Orifices surrounded by annulus at 34° to horizontal	19
‘Cone’	Smooth entry into orifices via 10° cone	8
‘Hourglass’	Smooth entry into orifices at varying angles	7–10 depending on volume of filling

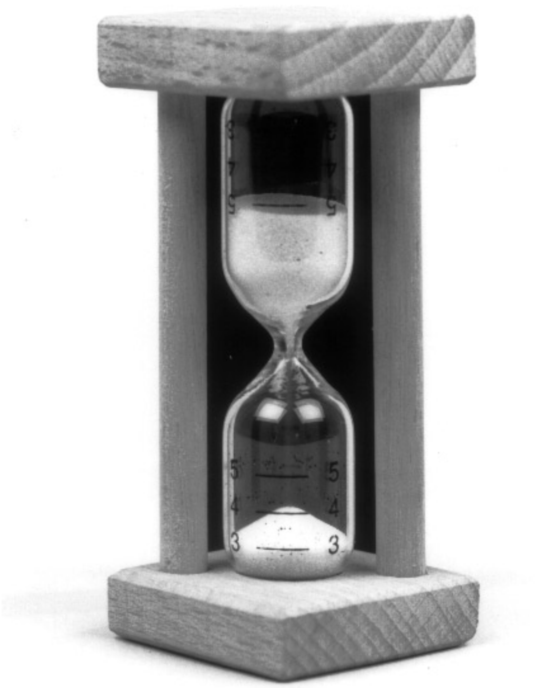

Figure 8. A modern egg-timer of cylindrical form.

7. Flow regimes

Two flow regimes may be distinguished: internal motions within the reservoir that deliver particles to the orifice, and the situation generated by the jet of particles that have left the orifice, and so are external to the reservoir.

7.1. External

Figure 1 shows how the stream of particles falling from the orifice builds up a conical pile upon the base of the lower vial. The angle of a diametral section with the horizontal constitutes the static angle of repose of the constituent particles; above this value downslope avalanching will occur to re-form the static angle and give a stable cone [22, 23]. The relevance of the sandpile to chaos theory has recently attracted argument [14].

Sharp and irregular materials with high intergranular interactions build narrower cones, with a higher angle of repose, than smooth spherical particles. The ballotini used here exhibited a static angle of repose of 25°.

7.2. Internal

Figure 1 also shows a conical crater formed in the upper vessel. In shape, it is close to being a mirror image of the lower pile, and is a result of the flow regime developed in the emptying reservoir of particulate material—a topic that intrigued distinguished physicists and engineers over a century ago [24, 25], and continues to do so [2, 14, 26–31].

Two main types of internal flow have been distinguished [11]:

(i) Plug flow, where the granules flow towards the outlet in a channel or cone formed within the material itself.

(ii) Mass flow, where the granular material behaves in a more uniform and liquid-like manner, with all the particles in motion and an upper horizontal surface continuously descending.

These are extremes; intermediate states are possible. Typically, mass flow reservoirs have smooth steep walls with the lower sections approaching a wide aperture at an angle greater than the angle of repose. The rate of delivery is comparatively high. Thus, the ‘standard funnel’ used in the preliminary tests tended to exhibit mass flow. Sandglasses, on the other hand, normally endeavour to measure intervals of time with a minimal volume of fine particles, necessitating low rates of flow through small apertures separating comparatively large reservoirs. Commonly, a horizontal shelf-like annulus of the diaphragm is exposed between the aperture and the glass walls. In these circumstances plug flow is likely, whatever the shape of the reservoir.

It may be noted that, once the crater in a plug-flow cylindrical vessel reaches the walls, the circle of contact should descend linearly with time until close to the aperture [11]. It is therefore reasonable for modern cylindrical egg-timers to be marked for 3, 4 and 5 minutes (figure 8): no great accuracy is expected. Fixed-period glasses should give a superior performance, and are the norm in historical examples.

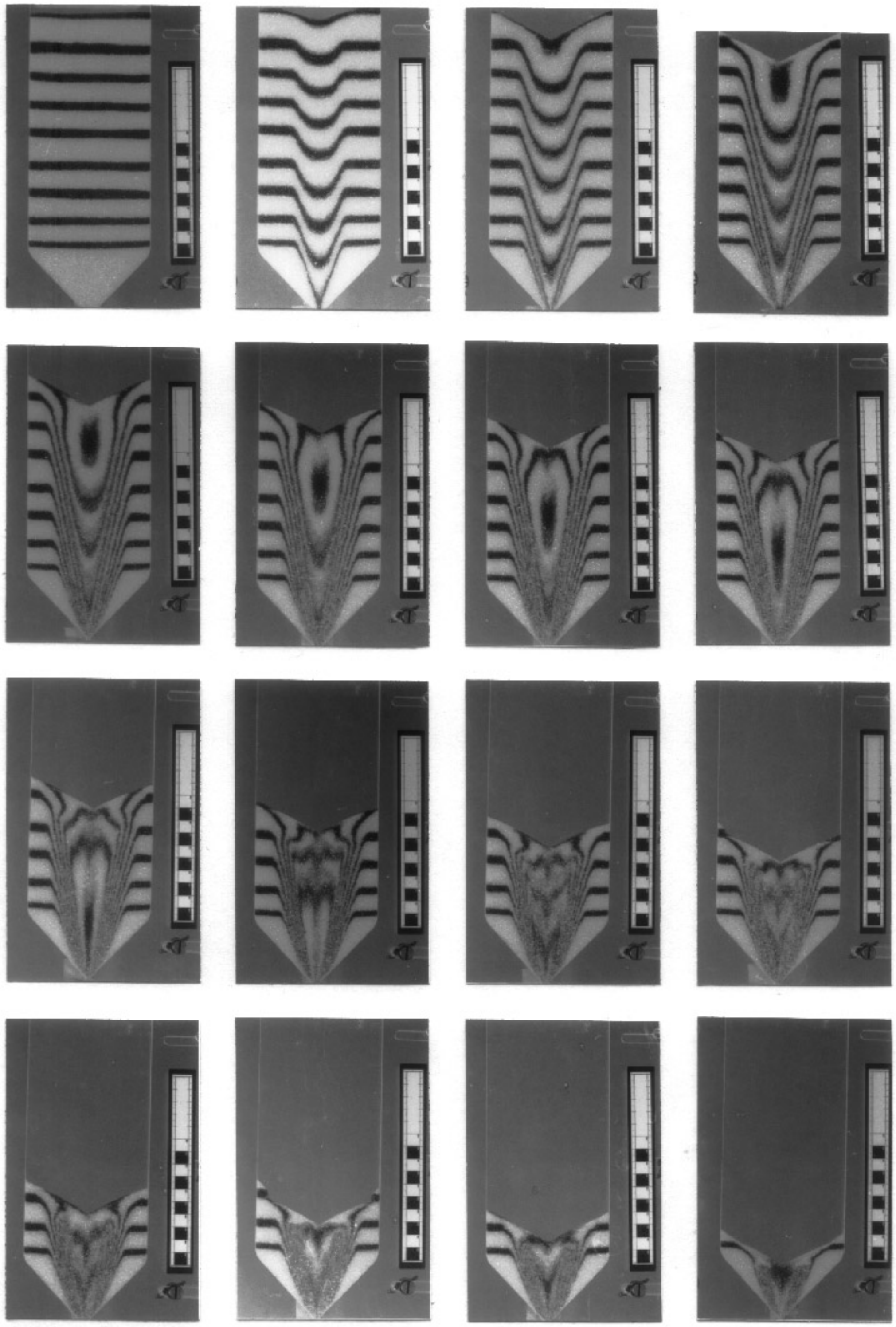

Figure 9. Plug-flow patterns within an undisturbed 'silo' reservoir. Scale in cm.

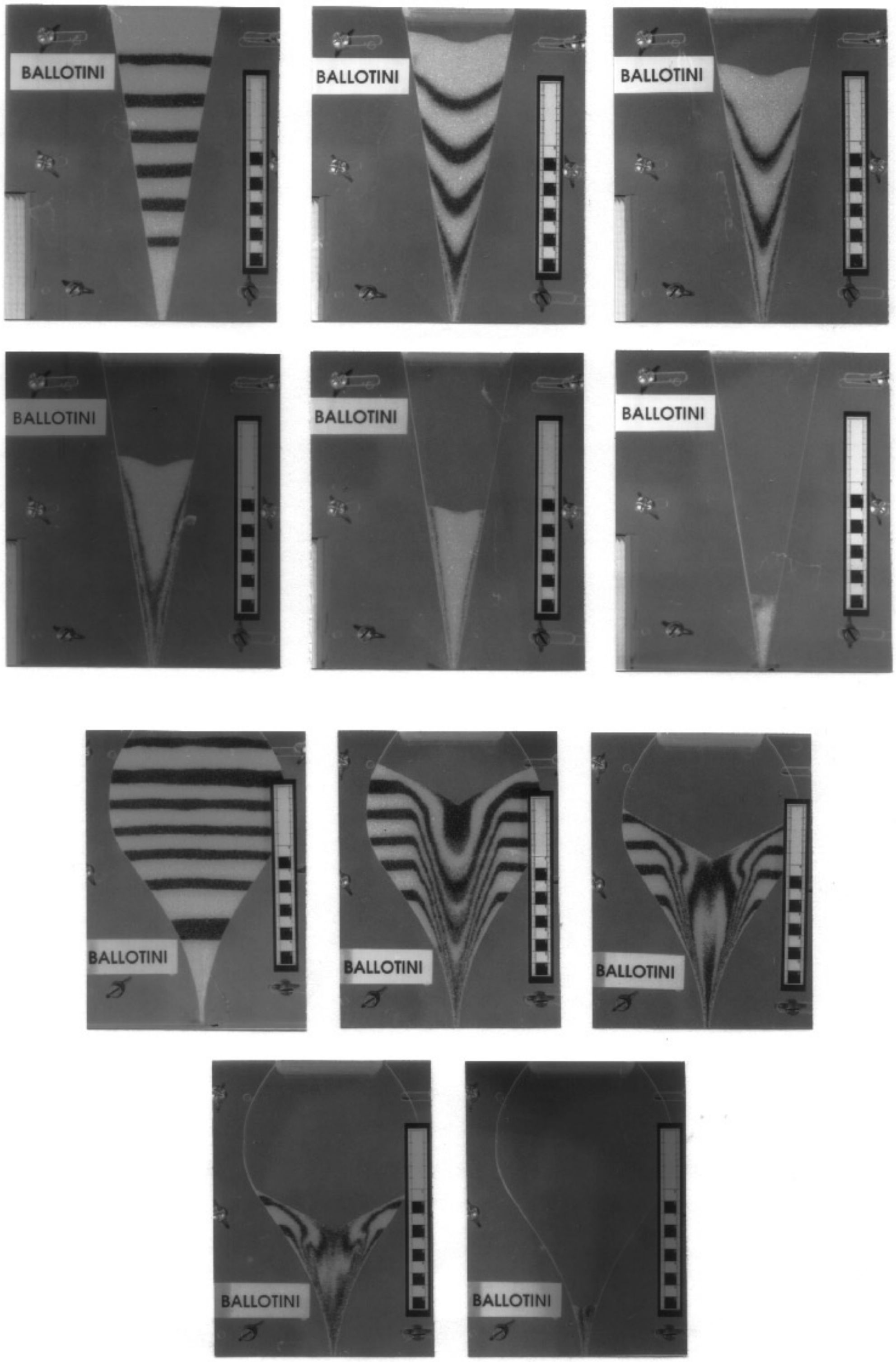

Figure 10. Plug-flow patterns within undisturbed ‘conical’ and ‘hourglass’ reservoirs. Scale in cm.

8. Model studies

Internal flow regimes were demonstrated by making demountable 'two-dimensional' models in perspex. Two central portions 8mm thick defined the shape of the reservoir, and were sandwiched between 260 mm squares of the same material. Clamping screws working in slots enabled the separation of the inner components to be adjusted, so varying the size of the (rectangular) aperture. Conical (14° semi-vertical angle) industrial silo, and traditional hourglass-shaped reservoirs were simulated. Polishing with a commercial anti-static fluid eliminated electrostatic adhesion to the walls.

A portion of the standard ballotini was coloured black with diluted cellulose lacquer, and then layered horizontally between the untreated white beads, the orifice being temporarily sealed with adhesive tape. The apparatus was supported vertically in a wooden stand. A photograph was taken, the orifice opened for a short period, and the result again photographed. This procedure was repeated several times with each shape, yielding the results shown in figures 9 and 10. It will be seen that a silo delivering under a slow plug-flow regime does not obey the rule 'last-in-last-out'. Plug flows only are illustrated here, but mass flow regimes could be obtained and pictured with wider apertures.

It may legitimately be questioned whether the 2-D models accurately reflect the behaviour of real 3-D reservoirs. We believe that they do because:

- The sequence of emptying periods obtained when all models were set to the same width of aperture, and loaded with the same mass of ballotini, was the same as that observed with the 3-D glass vessels above.
- Our results for the silo shape agree with those found by other investigators using similar, [26] x-ray, [2] and gel-sectioning [13] methods.
- The 'stop–start' procedure we employed to obtain photographic records gave the same integrated period as continuous flow from the same model. Tuzun *et al* [29] found the same.

9. Calibration and reproducibility

In view of the many uncertainties involved, the calibration procedure used in the bulk manufacture of fixed period sandglasses in the 17th century can hardly be bettered. This was to overfill a batch of glasses, allow them to run against a standard glass, and then at the termination of the desired period to turn all on their sides. Excess sand was then removed from the erstwhile upper reservoir, and the ampoules sealed up.

It was observed that, although slower running, the form of orifice consisting of a circular hole surrounded by a horizontal annulus (as in our 'bottle' apparatus) resulted in a more repoducible emptying period than those designs where an all-glass reservoir curved-in steplessly to the exit aperture. Thus, in one series of trials, a 2.8 mm annular aperture gave a mean period of 19 m 45 s with a standard deviation of $\pm 5\,\mathrm{s}(\pm 0.4\%)$, while the continuously tapering hourglass and cone shapes could give standard deviations up to $\pm 1.5\%$. It is thought that this might be due to the fact that possibly variable interactions with the walls of the container are minimized with the annular form of orifice, where in plug flow the moving particles are guided entirely by similar particles.

10. Environmental factors

Finally, factors external to a sealed sandglass were investigated.

10.1. Disturbance

It has been suggested [11] that, in addition to its immunity to freezing, the sandglass was perhaps first used on ships because of a lesser susceptibility to motion than clepsydrae and early mechanical clocks. Indeed, a number of applicants proposed methods based on a sandglass to 'discover the longitude' [32].

To investigate the effect of movement and disturbance on a sandglass, the 'bottle' fitted with a 3.6 mm diameter aperture was continually moved about manually (but not shaken) while the ballotini were running. Its mean period of 11 m 31 s when undisturbed was increased to 11 m 35 s. It was noticeable that the upper surface of the ballotini in the moving reservoir remained horizontal throughout, being unable to develop a crater. It appears that disturbance results in elimination of the delicate internal flow pattern characterizing plug flow, supplanting it by a slower mass flow regime. Continuous thumping of the bottle with the heel of the hand increased the emptying period to 11 m 54 s.

10.2. Temperature

The effect of temperature on the sandglass would be expected to be small, expansion of the hole with increase in temperature being foreseen as the only significant possiblity. Even so, it could well be lost in the normal variation.

To check on this, a modern all-glass egg-timer was operated a number of times submerged in a beaker of boiling water. No difference in its mean period from that displayed at ambient temperature could be detected. We conclude that the rumoured expedient of sailors placing the sandglass beneath their jackets on a cold winter night ('Warming the Bell') would provide no more than psychological benefit.

Summary

(i) The traditional sandglass consists of two glass ampoules joined by narrow tapering necks or, in early examples, separated by a thin metal diaphragm pierced with a small central hole.

(ii) The device constitutes an interval timer, the period being measured by the time elapsing between the inversion of the apparatus and the last grains leaving the upper reservoir. Although commonly called an hourglass, other periods (such as 1/4, 1/2 and 3/4 hour) were frequently delineated, so the term sandglass is preferred here.

(iii) Unlike the ancient water-operated clepsydra, the sandglass is known only from medieval times. This is especially surprising when it is recalled that it is not liable to freezing.

(iv) Historical sandglasses were always sealed as tightly as the technology of the time allowed, for moisture seeping in could cause caking of the filling and consequent irregular running. Air displaced by the descending grains must therefore rise as a countercurrent and percolate through the particulate bed in the upper reservoir.

(v) The particles in a sandglass need not be silica sand, but should always be clean, dry, and sieved to be of closely similar size.

(vi) Particles of a smooth spherical shape are best, promoting reproducibility even at low rates of flow. This requirement is most conveniently met nowadays by the tiny glass beads available in bulk as 'ballotini'.

(vii) To give a smooth and consistent flow without occasional partial or complete blockage, the aperture through which the ballotini fall should be at least $4\times$ their maximum diameter. (For particulate material in general, $5\times$ the maximum grain dimension would be safer.)

(viii) The rate of flow is independent of height in the reservoir, except over the last few cm.

(ix) The emptying period is independent of the density of the particulate filling, provided that its total volume is the same and the shape, size and surface characteristics of the grains are identical.

(x) The period P delineated by a sandglass is controlled by the shape of the vessels from which it is made, the diameter of the aperture separating them, and the nature and volume of the particulate material sealed within it.

(xi) For spherical ballotini passing a sieve of maximum aperture d mm, it was found that:

$$P \propto \frac{V}{(D-d)^{2.5}}$$

where V represents the volume of ballotini in ml and D is the diameter of a circular orifice in mm. (Alternatively, a value of d given by $1.3\times$ the mean bead diameter may be employed.)

(xii) The shape of the reservoir containing the ballotini is reflected in the constant of proportionality to be applied to the above expression to give P in seconds. For 'hourglass', 'cone', 'silo' and 'annular shelf' designs K is 7–10, 8, 19 and 21 respectively.

(xiii) The above characteristics are the result of internal flow regimes in the particulate filling that are quite different from the flow of true liquids. Sandglasses are normally required to be slow-running and therefore exhibit 'plug-flow'—a situation where a continuously collapsing arch over the exit aperture generates a channel above it that is continually replenished by the flow of particles from the sides. A circular crater is thereby generated in the upper reservoir, while a matching cone grows in the receiver. Their angles are controlled by the 'angles of repose' of the particles (25° for ballotini). Internal flow patterns in reservoirs of various shapes have been illustrated by sequential photographs of 2-D perspex models containing marker bands of coloured ballotini.

(xiv) The reproducibility of a 19 m 45 s glass containing a 'traditional' annular orifice 2.8 mm in diameter was found to be ± 5 s($\pm 0.4\%$) if it was allowed to run without disturbance.

(xv) The period of a sandglass was lengthened by any disturbance, but was essentially independent of temperature up to 100 °C.

References

[1] Mills A A 1982 Newton's water clocks and the fluid mechanics of clepsydrae notes and records *R. Soc. Lond.* **37** 35–61

[2] Lee J, Cowin S C and Templeton J S 1974 An experimental study of the kinematics of flow through hoppers *Trans. Soc. Rheo.* **18** 247–69

[3] Bagnold R A 1941 *Physics of Blown Sand* (London: Methuen)

[4] Vivielle 1934 Sandglasses, clocks of the sea *Bull. Officiel du Yacht Club de France* Transl. R T Balmer and published 1977 *Bull. Nat. Assn. Watch and Clock Collectors (USA)* **19** 1–19

[5] Waters D W 1955 Early time and distance measurement at sea *J. Inst. Nav.* **8** 153–73

[6] Waters D W 1958 *The Art of Navigation in England in Elizabethan and Early Stuart Times* (London: Hollis and Carter)

[7] Jünger E 1954 *Das Sanduhrbuch* (Frankfurt)

[8] Balmer R T 1979 The invention of the sand clock *Endeavour* **3** 118–22

[9] Turner A J 1982 'The accomplishment of many years': Three notes towards a history of the sand-glass *Ann. Sci.* **39** 161–72

[10] Turner A J 1984 *Water clocks, sand glasses, fire clocks* Vol 1, Part 3 of a catalogue of the Time Museum, Rockford, Illinois (Rockford, published by the museum)

[11] Balmer R T 1978 The operation of sand clocks and their medieval development *Tech. Culture* **19** 615–32

[12] Möller B 1992 Sanduhren Ansehen Besitzen Selbst anfertigen *Uhren* (Germany) **2** 25–31

[13] James C A, Yates T A, Walford M E R and Gibbs D F 1993 The hourglass *Phys. Ed.* **28** 117–20

[14] Mehta A (ed) 1994 *Granular Matter: An Interdisciplinary Approach* (London: Springer)

[15] Jaeger H M and Nagel S R 1992 Physics of the granular state *Science* **255** 1523–31

[16] Drover C B, Sabine P A, Tyler C and Coole P G 1960 Sand-glass 'sand': Historical, analytical and practical *Antiq. Horol.* **3** 62–72

[17] English Glass Co. Ltd, Scudamore Road, Leicester, UK
[18] Cassidy T 1955 Hour-glass facts *Horol. J.* October 658
[19] Mills A A 1988 The mercury clock of the 'Libros del Saber' *Ann. Sci.* **45** 329–44
[20] Franklin F C and Johanson L N 1955 *Chem. Eng. Sci.* **4** 119–29
[21] Beverloo, W A, Leniger H A and van de Velde J 1961 *Chem. Eng. Sci.* **15** 260–9
[22] Carrigy M A 1970 Experiments on the angle of repose of granular materials *Sedimentology* **14** 147–58
[23] Allen J R L 1970 The avalanching of granular solids on dune and similar slopes *J. Geol.* **78** 326–51
[24] Reynolds O 1885 On the dilatancy of media composed of rigid particles in contact *Phil. Mag.* **20** 469–81
[25] Rayleigh, Lord 1906 Note on the principle of the sand-clock. Addendum to 'On an Instrument for Compounding Vibrations . . . ' *Phil. Mag.* **11** 130
[26] Brown R L and Hawksley P G W 1947 The internal flow of granular masses *Fuel* **27** 159–73
[27] Brown R L and Richards J C 1970 *Principles of Powder Mechanics* (New York: Pergamon)
[28] Wieghardt K 1975 Experiments in granular flow *Ann. Rev. Fluid Mechanics Palo Alto, CA* ed M van Dyke, W G Vincent and J V Wehausen pp 89–114
[29] Nedderman R M, Tüzün U, Savage S B and Houlsby G T 1982 The Flow of Granular Materials. I: Discharge Rates From Hoppers *Chem. Eng. Sci.* **37** 1597–609
[30] Tüzün U, Houlsby G T, Nedderman R M and Savage S B 1982 The flow of granular materials. II: Velocity distributions in slow flow *Chem. Eng. Sci.* **37** 1691–709
[31] Behringer R P 1995 Mixed predictions *Nature* **374** 15
[32] Baillie G H 1951 *Clocks and Watches: An Historical Bibliography* (London: NAG)